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## Testing self-organized criticality in the crust using entropy: A regionalized study of the CMT global earthquake catalogue

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[1] We test the notion of self-organized criticality (SOC) and the proximity to the critical point in the brittle crust. If the system were strictly critical, we would expect an infinite correlation length with minimal temporal or spatial predictability. An alternative view is that the Earth is in a self-organized subcritical (SOSC) state with a finite and systematically fluctuating correlation length. This would imply a system that is sufficiently near-critical to maintain power law scaling relations over a finite scale range, but can intermittently reach criticality in the form of a single large earthquake when the correlation length becomes effectively infinite over the scale of the observed region. Here we address the question of proximity to criticality from the viewpoint of statistical physics by describing a regionalized study equivalent to the ensemble approach in thermodynamics. Flinn-Engdahl regionalization of global seismicity is used to calculate the expectation of the logarithm of energy  $\langle \ln E \rangle$  and entropy  $S$  from centroid moment tensor (CMT) data for different seismic regions. We compare a phase diagram for  $S$  and  $\langle \ln E \rangle$  from the data with an analytical statistical mechanical solution and find that they are in good agreement. The analysis shows systematic spatial heterogeneity in entropy that is associated with the tectonic deformation style. Oceanic ridges are seen to be low entropy (relatively ordered), and subduction zones have higher entropy (less ordered) with collision-zones scattered at the two extremes. Statistically resolvable phase variations in the system as a whole point toward it being better described as subcritical in the spatial ensemble.

**INDEX TERMS:** 3220 Mathematical Geophysics: Nonlinear dynamics; 7209 Seismology: Earthquake dynamics and mechanics; 7223 Seismology: Seismic hazard assessment and prediction;

**KEYWORDS:** self-organized criticality, statistical mechanics, entropy, spatial predictability

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### 1. Introduction

[2] The energy radiated by earthquakes usually is described by the Gutenberg-Richter (G-R) law [Gutenberg and Richter, 1954],

$$\log[N(m)] = a - bm, \quad (1)$$

where  $N(m)$  is the number of earthquakes with magnitude  $\geq m$ ,  $a$  and  $b$  are constants with  $b \sim 1$ , and  $m$  is a logarithmic measure of radiated seismic energy. This law (1) breaks down at larger magnitudes so is more generally expressed by a “gamma” distribution of energy which is similar to the G-R law but with an exponential tail (see review by Main [1996]) in the form

$$p(E) \sim E^{-B-1} \exp(-E/\theta), \quad (2)$$

where  $p(E)$  is the probability density function for having an earthquake of energy  $E$ ,  $B$  is the earthquake scaling exponent

(proportional to  $b$  above) found globally to be  $\sim 2/3$  [Kagan, 1997, 1999; Godano and Pingue, 2000; Leonard *et al.*, 2001], and  $\theta$  is a measure of the maximum event size. The power law or “fractal” component of this distribution suggests that the Earth belongs to a dynamic class of systems described as being in a state of self-organized criticality (SOC) [Bak and Tang, 1989; Olami *et al.*, 1992]. The original model of SOC of Bak, Tang, and Wiesenfeld [Bak *et al.*, 1987] (referred to here as the BTW model) was conceived from a numerical cellular automaton based on the Burridge-Knopoff model [Burridge and Knopoff, 1967]. The model self-organizes to produce a power law in the frequency-size distribution of clusters or avalanches similar to the Gutenberg-Richter law, despite having very simple rules governing neighbor-neighbor interactions between cells, and no “tuning” parameters. Although the definitions of SOC are general, qualitative, and often not very clear in the literature [Sornette *et al.*, 1990; Malamud and Turcotte, 1999], a pure state of SOC as suggested from the BTW model would imply [Bak *et al.*, 1987; Jensen, 1998] (1) a slowly driven system far from equilibrium with small fluctuations about the critical state over large timescales; (2) sensitivity to minor perturbations that could trigger large

events (avalanches) that can span the length scale of the system ( $\theta \rightarrow \infty$ ); and (3) a power law nature that is global and independent of local dynamics. In the case of the Earth, SOC would therefore require the crust to be perpetually near a state of global failure, rendering individual events unpredictable both temporally and spatially.

[3] An alternative school of thought suggests that the Earth's crust is instead in a self-organized "subcritical" state (referred to here as SOSC). SOSC is characterized by the system being below the critical point (with a finite correlation length) but still maintaining power law statistics on a local scale. Observations consistent with SOSC include a finite correlation length inferred from the gamma distribution observed in the earthquake frequency-magnitude relation [Kagan, 1997, 1999; Leonard *et al.*, 2001], anomalous stress diffusion in the crust during earthquake triggering [Marsan *et al.*, 2000; Huc and Main, 2003], and accelerated seismicity before large events [e.g., Bowman *et al.*, 1998; Robinson, 2000; Zöller *et al.*, 2001]. A SOSC state would also suggest a finite degree of statistical predictability in the dynamics of the system, contrary to pure SOC. It is therefore important to address the "criticality question" by conceiving formal methods of assessing or quantifying the proximity of the Earth's crust to criticality. The outcome will have implications for our current approach in addressing earthquake population predictability ([http://www.nature.com/nature/debates/earthquake/quake\\_frameset.html](http://www.nature.com/nature/debates/earthquake/quake_frameset.html)).

[4] To address the criticality question, Main and Al-Kindy [2002] revert to using the fundamental tools of statistical physics to examine proximity to criticality in the crust by measuring the change in entropy  $S$ , with seismic energy release  $E$  in annual global data from the Harvard centroid moment tensor (CMT) catalogue. With reference to an analytical model, the temporal ensemble in the crust was found to be very near a critical state on a global scale, with fluctuations of entropy of the order of  $\pm 10\%$ . It was not established, however, if fluctuations of 10% would be sufficient to conclude a SOC state in the temporal ensemble. Also, any regional variations in the spatial ensemble would be smoothed out as data were binned in time rather than in space. The use of established theories of statistical physics, and the fundamentals of thermodynamics, nonetheless is a physical, well-founded, and reproducible approach to quantifying the proximity to criticality in the crust. Although other studies have also used entropy to understand earthquake properties [e.g., Goltz, 1997; Goltz and Böse, 2002; Nicholson *et al.*, 2000; Posadas *et al.*, 2002], they are based on a configurational entropy that concentrates on the spatial properties of organization and clustering with scale rather than the dynamical aspects of energy fluctuations investigated here.

[5] In this paper we extend the global temporal analysis of Main and Al-Kindy [2002] to a spatial ensemble approach by analyzing data from the CMT catalogue using Flinn-Engdahl (FE) regionalization. Such an approach is common in physics in the analysis of ergodic systems where an individual system can exchange energy with an external sink (i.e., a "canonical ensemble"). The main purpose of this study is to establish if statistically significant regional variations in the entropy-energy relation exist and, if so, to assess if the variations are more indicative of a SOC or SOSC system. If statistically significant regional variations are found, these

would contradict strict SOC and indicate a subcritical state (SOSC).

## 2. Theory

### 2.1. Analytical Solution

[6] The basic principle of statistical physics is that macroscopic properties of a system can be predicted when the probability distribution of the energetic microstates of the system are known. The probability distribution is determined by the maximum entropy configuration, given constraints based on what is known about the system. If the microscopic probability of the microstates in an increment  $dE$  is  $p(E)dE$ , the information entropy  $S$  [Shannon, 1948; Jaynes, 1957] is defined by

$$S = - \int_{E_{\min}}^{E_{\max}} p(E) \ln p(E) dE, \quad (3)$$

where  $E_{\min}$  and  $E_{\max}$  are the minimum and maximum possible energy states. Jaynes [1957] showed that this definition is identical to the thermodynamic entropy when the variable is energy. This definition of the entropy follows from the requirement in statistical inference to have a function that is positive, increases with increasing uncertainty, and is additive for independent sources of uncertainty (see the appendix to Jaynes [1957]). We solve for  $p(E)$  by maximizing the entropy subject to the fundamental probabilistic constraint

$$\int_{E_{\min}}^{E_{\max}} p(E) dE = 1 \quad (4)$$

and  $N$  constraints

$$\langle f_i(E) \rangle = \int_{E_{\min}}^{E_{\max}} f_i(E) p(E) dE; \quad i = 1, \dots, N \quad (5)$$

where  $f_i$  are functions of the energy. The maximum entropy distribution is the most likely, subject to what is known [Jaynes, 1957]. We therefore look for  $\delta S = 0$ , using the method of Lagrangian undetermined multipliers, subject to equations (4) and (5), whence

$$p(E) dE = e^{-\lambda_0 - \lambda_1 f_1(E) - \lambda_2 f_2(E) - \dots - \lambda_N f_N(E)} dE. \quad (6)$$

The constants  $\lambda_i$  are Lagrangian undetermined multipliers, to be determined by substituting equation (6) into (4) and (5). If we define

$$\lambda_0 = \ln Z, \quad (7)$$

then

$$Z = \int_{E_{\min}}^{E_{\max}} e^{-\lambda_1 f_1(E) - \lambda_2 f_2(E) - \dots - \lambda_N f_N(E)} dE, \quad (8)$$

$$\langle f_i(E) \rangle = -\partial \ln Z / \partial \lambda_i. \quad (9)$$

The function  $Z$  is a normalizing constant for unit total probability and is known as the partition function. Once its

form is known, all of the other macroscopic variables  $f_i$  can be calculated from its partial derivative using equation (9).

[7] For example, let us take the constraint of finite total system energy  $f_1(E) = E$ , and finite natural log energy  $f_2(E) = \ln(a_2 E)$  (the constant  $a_2$  will be defined below), whence

$$p(E)dE = \frac{e^{-\lambda_1 E - \lambda_2 \ln(a_2 E)}}{Z} dE. \quad (10)$$

Let us define  $\theta = \lambda_1^{-1}$ ,  $B + 1 = \lambda_2$ , and  $a_2 = E_0^{-B}$ , so that

$$p(E)dE = \frac{E_0^B E^{-B-1} e^{-E/\theta}}{Z} dE. \quad (11)$$

This form of the distribution (the exact form of equation (2)) was derived using the above method by *Shen and Mansinha* [1983] and *Main and Burton* [1984] and has been more recently applied to earthquake frequency-moment data extensively by *Kagan* [1997, 1999] and *Leonard et al.* [2001]. It is a mixture of a power law part at small energies and a Boltzmann exponential with a “temperature term”  $\theta$  at large energies. In the case of earthquakes, the exponent  $B$  of the power law part is proportional to the  $b$  value of the Gutenberg-Richter frequency-magnitude relationship, and the “tectonic temperature”  $\theta$  is a measure of the maximum event size. For instruments acting as velocity transducers,  $B = 2/3b$  [e.g., *Turcotte*, 1997], if  $b$  is the slope of the frequency-magnitude relation (equation (1)). The power law term can be regarded as due to the increasing degeneracy  $E^{-B-1}$  of the potential energy transitions as we go to smaller energies. A smaller energy release requires a smaller source area, and hence more numerous potential sites in a given strained volume [*Kanamori and Anderson*, 1975; *Main and Burton*, 1984]. Hence  $f_2$  here is equivalent to a geometric constraint on the mean source area: Small events are more likely since there are more ways they can fit into a given volume [*Rundle*, 1993].

[8] In a system with infinite  $E_{\max}$ ,  $\theta$  is constrained to be positive to preserve finite  $\langle E \rangle$ . However, if  $E_{\max}$  is finite, then  $\theta^{-1} = 0$  or  $\theta^{-1} < 0$  can occur. *Dahmen et al.* [1998] and *Main et al.* [2000] showed that the sign of  $\theta$  could be used to indicate two separate phases on either side of a critical point. A subcritical state is defined by positive  $\theta$  (energy states with high energy levels are less likely to be occupied, the normal case in thermodynamics), a critical state by  $\theta^{-1} = 0$  (energy states have an equiprobable distribution) and a supercritical configuration (with a population inversion at high energy states) with negative  $\theta$ . The supercritical state can be identified with the “characteristic earthquake” model of *Schwartz and Coppersmith* [1984]. It should be noted, though, that “no system is likely to survive long in a supercritical state” [*Vere-Jones*, 1976, 716].

[9] From equations (3) and (11) using logarithmic bins to define  $S$ , the entropy is [*Main and Al-Kindy*, 2002]

$$S = \ln Z + B \langle \ln(E/E_0) \rangle + \langle E \rangle / \theta. \quad (12)$$

The logarithmic bins are necessary when examining real data to obtain stable estimates of  $S$ . For linear bin intervals

in energy, we would obtain the same relation with  $B$  in equation (12) replaced with  $B + 1$ . Note that since the Shannon entropy contains no Boltzmann prefactor  $k$ , the entropy here is dimensionless (in the Boltzmann distribution  $\theta = kT$ , where  $T$  is the thermodynamic temperature). Another implicit assumption in using expectation values as constraints is that within the time period of the data being used to calculate an individual value of  $Z$ ,  $\langle E \rangle$ ,  $\langle \ln(E/E_0) \rangle$ , or  $S$ , the probability distributions are stationary, i.e., they are characterized by statistical properties that do not change over large periods of time. Recently it has been shown that entropy in far-from-equilibrium steady state systems such as SOC can be treated in the same way as equilibrium thermodynamic systems [*Dewar*, 2003], further justifying the theoretical approach above.

[10] From equation (12), if we are near the critical point, where  $\theta \rightarrow \infty$ , we would expect a relation of the form

$$S = \Lambda + B \langle \ln E \rangle, \quad (13)$$

where

$$\Lambda = \ln Z - B \ln E_0. \quad (14)$$

There are then three ways of distinguishing a system from the critical point: (1) systematic, statistically significant fluctuations in  $S$ , (2) a linear slope  $\delta S / \delta \langle \ln E \rangle \neq B$ , and (3) any statistically significant curvature (nonlinearity) in the relation between  $S$  and  $\langle \ln E \rangle$ . These are simple and verifiable criteria for distinguishing between a state of SOC and intermittent criticality or SOSOC.

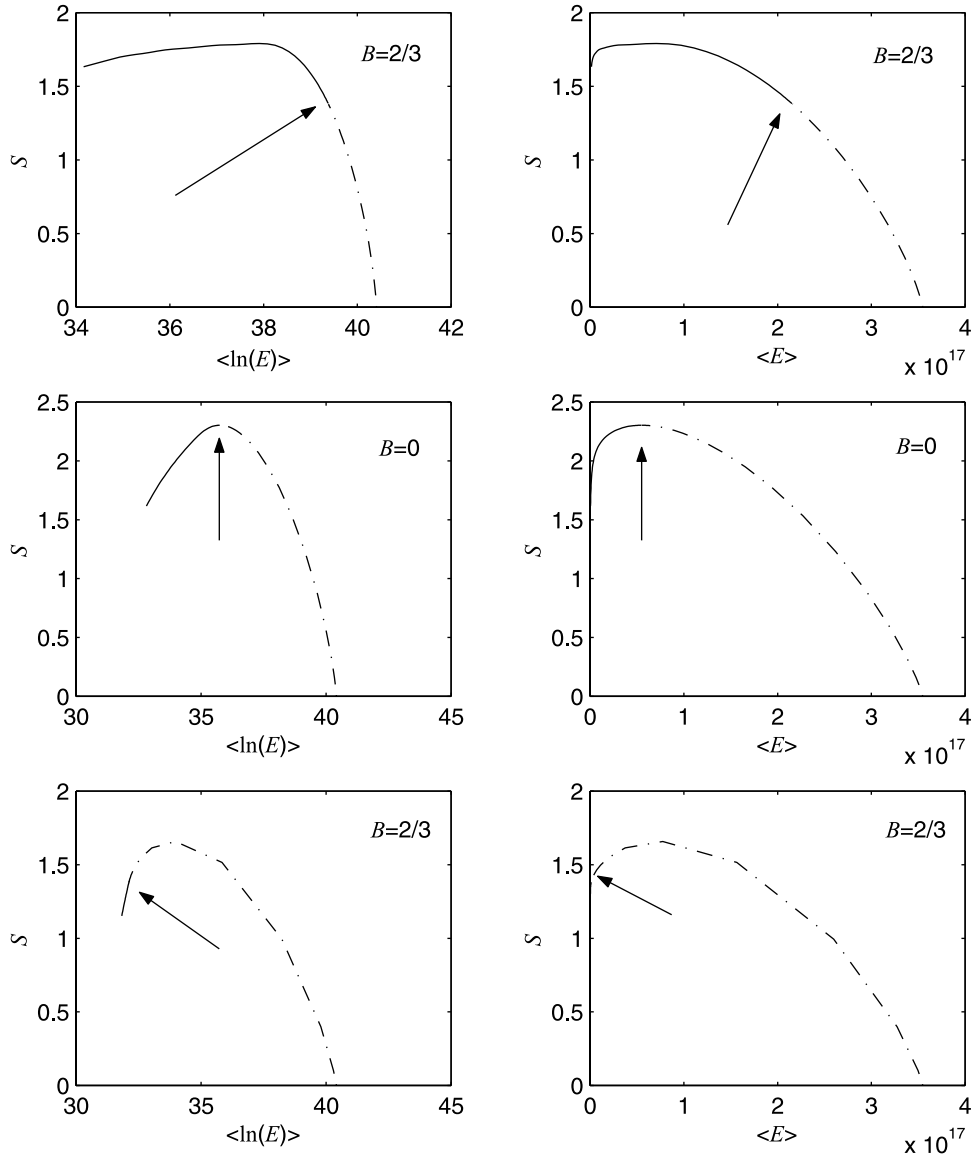
[11] At this point we should note that we have implicitly assumed that  $\langle \ln E \rangle$  and  $\langle E \rangle$  are independent, because the constraint  $\langle \ln E \rangle$  is geometric, whereas the constraint  $\langle E \rangle$  is energetic. Although it might be regarded as counterintuitive, this assumption has been validated independently by the negligible correlation observed between temporal variations in  $\langle \ln E \rangle$  and  $\langle E \rangle$  using global earthquake data [*Main and Al-Kindy*, 2002]. This is consistent with the general formalism introduced by *Jaynes* [1957] in equations (3)–(9) above. Other thermodynamic parameters can be obtained once the entropy is known. For example, from equation (12)

$$\theta^{-1} \sim \partial S / \partial \langle E \rangle. \quad (15)$$

This relation demonstrates formally that  $\theta$  is an equivalent temperature term for the system.

[12] There is an important caveat to the application of thermodynamics to earthquake statistics. The thermodynamic formulation is strictly based on an internal energy  $U = \langle E \rangle$ , but it is not possible to determine  $U$  for the earthquake problem, since we do not have independent information on the strain energy distribution in the Earth because we cannot independently measure the stress. Therefore applications of thermodynamics to earthquake systems have assumed, explicitly or implicitly, that the distribution of radiated energy is related to that of the internal energy [e.g., *Rundle*, 1993; *Dahmen et al.*, 1998; *Main and Al-Kindy*, 2002]. This implies that large earthquakes are more likely to occur in time periods when the internal strain energy is also high. This assumption cannot be validated for the Earth, but has been shown to be consistent with





**Figure 1.** Theoretical curves of  $S$  against  $\langle \ln(E) \rangle$  and  $\langle E \rangle$  with  $B = -2/3$ , 0, and  $2/3$  for subcritical (solid line), critical (arrow), and supercritical (dashed line) regimes created using equations (3) and (5).

numerical models for earthquakes as complex interacting systems [Rundle *et al.*, 1995; Main *et al.*, 2000].

## 2.2. Analytical Predictions

[13] In order to illustrate predictions of the analytical theory outlined above, synthetic curves are created to show the energy-entropy phase space for different values of  $B$  for subcritical ( $\theta > 0$ ), critical ( $\theta = \infty$ ), and supercritical ( $\theta < 0$ ) regimes (Figure 1). The curves were constructed using equations (3)–(11) substituting the parameters with numerical values similar to those found from CMT data analyzed below. Using the relations [Kanamori and Anderson, 1975]

$$E = 10^{1.5m+4.8}, \quad (16)$$

$$\theta = 10^{1.5m_0+4.8}, \quad (17)$$

the energy  $E$  corresponding to theoretical magnitudes  $m$  in the range  $5.5 < m < 8.5$  was calculated in accordance with

available data. The “temperature” term  $\theta$  was also calculated for magnitudes  $m_\theta$  in the range  $7 < m_\theta < \infty$  for both positive (subcritical) and negative (supercritical) values of  $\theta$ . Using these values,  $p(E)$  was then calculated using equation (11) for  $B = -2/3$ , 0,  $2/3$  as well as the corresponding values of energies  $\langle E \rangle$  and  $\langle \ln E \rangle$ , and  $S$  using equations (5) and (3), respectively. Figure 1 shows the theoretical values of  $\langle E \rangle$  and  $\langle \ln E \rangle$  and corresponding  $S$  for  $B = -2/3$ , 0, and  $2/3$  with the critical point indicated by the arrow, subcritical (solid line), and supercritical (dashed line) regions in the energy-entropy phase spaces. It can be seen from Figure 1 that only for the case  $B = 0$  (no degeneracy) that the critical point occurs at the point of maximum possible entropy for both  $\langle \ln E \rangle$  and  $\langle E \rangle$ . In the case of  $B = 2/3$ , the critical point is below the point of maximum entropy along the  $x$ -axis and the curve is similar to that observed for real earthquake populations [Main and Al-Kindy, 2002]. The opposite is true for the  $B = -2/3$ .

For  $B = -2/3$  and  $B = 2/3$ , the global maximum entropy ( $S_{\max} \sim 1.7$ ) is lower than the maximum possible entropy for  $B = 0$  ( $S_{\max} \sim 2.3$ ) due to the effect of the degeneracy term. It can also be seen from the plots of  $S$  against  $\langle \ln E \rangle$  that equation (13) does apply at the critical point, indicated by the arrows in the diagrams. Thus any curvature in the relation between  $S$  and  $\langle \ln E \rangle$ , or deviations in the underlying slope  $B$ , would imply fluctuations away from the critical state.

### 3. Data Analysis

[14] We investigate the criticality question in two ways. First, we calculate the regional properties such as energy, entropy, and  $B$  and investigate if statistically significant variations occur between the various properties for different tectonic settings. Second, we look at the energy-entropy phase space and investigate, with reference to equation (13), if the phase space is more indicative of a SOC or SOSC system. In this study we used data from the Harvard centroid moment tensor (CMT) catalogue (<http://www.seismology.harvard.edu/>). The data used are for the time period 1977–2000 inclusive for all energies corresponding to scalar moment-magnitudes  $M \geq 5.5$  and for all depth ranges available, giving a total of 9042 events. The scalar moment magnitudes were converted to energy (Joules) using standard relationships [Kanamori and Anderson, 1975]. The Flinn-Engdahl (FE) regionalization of data was done using modified code acquired from the United States Geological Survey (<ftp://ghftp.cr.usgs.gov>) dividing the Earth in to 50 “tectonic” zones. The data, for regional comparison, were broadly grouped in to four main tectonic deformation styles following the classification of Kagan [1997]. These are subduction zones, collision zones, intracontinental zones, and mid-ocean ridges. The advantage of FE regionalization is that it predates the establishment of the CMT catalogue, and therefore any systematic differences between regions cannot be accredited to retrospective data or region selection bias.

[15] The energy-probability distributions for each zone were calculated by binning data in to bins with widths equivalent to a magnitude bin width  $\delta m = 0.25$ . This was the highest resolution possible while insuring there were no empty bins that could destabilize the calculations of  $S$ . The  $p(E)$  was normalized such that  $\sum p(E) = 1$  with the limits  $E_{\min} \leq E \leq E_{\max}$  and  $S$  for each zone was then calculated from  $p(E)$  using equation (3). The  $\langle \ln E \rangle$  and  $\langle E \rangle$  were calculated taking the means of  $\ln E$  and  $E$ , respectively, of all data available within a given FE region. In the analysis, any regions containing fewer than 30 seismic events were not used, as they are considered not to contain a sufficient number of events to stably calculate  $S$  and  $B$ , leaving a total of 7689 events spread over 32 zones.

## 4. Results

### 4.1. Regional Variations

[16] Table 1 gives all variables calculated for each FE zone. The  $B$  exponents were calculated using a least squares fit on the linear part of the power law (LSF) distributions on a log-log scale, and the  $b$  values were calculated using a

maximum likelihood fit (MLF) for comparison using [Aki, 1965]:

$$b = \frac{\log_{10} e}{\langle m \rangle - m_c}, \quad (18)$$

were  $\langle m \rangle$  is the mean magnitude and  $m_c$  is the catalogue magnitude threshold for complete reporting (the minimum value of  $m$  in the population). The phase variations between zones are shown in Figure 2 where the values of  $S$  against  $\langle \ln E \rangle$  lie along a curve. The relative statistical significance of the best line fit is discussed in section 4.3. Table 1 also shows that subduction zones are clustered at the higher end of the phase curve ( $\langle S \rangle = 1.1$ ) and mid-ocean ridges at the lower end ( $\langle S \rangle = 0.88$ ) with the collision zones scattered between the two, and also constituting the extreme maximum and minimum points. The entropy variation between the deformation styles is somewhat expected since mid-ocean ridges are along well-defined (organized) zones and therefore earthquake epicenters are clustered differently compared to subduction zones [Nicholson *et al.*, 2000]. Looking at this from an “information” point of view, we can say that we are more certain of what will occur in an ocean ridge than, say, along a subduction zone. This is because we are more certain (lower entropy) that large energy fluctuations (great earthquakes) will not occur. The difference is also represented in the higher  $b$  and  $B$  values for the mid-ocean ridges ( $\langle b \rangle = 1.24$ ,  $\langle B \rangle = 0.81$ ), consistent with the results of Kagan [1997], and also implying a more clustered (ordered) distribution of events.

[17] In equation (13),  $B$  is a constant so variations in  $S$  are assumed to be due to changes in  $\langle \ln E \rangle$ . Testing the relationship between  $B$  and  $b$  with  $S$ , we see that there is a strong correlation between them (Figure 3a), suggesting that the scaling exponents  $b$  and  $B$  can be used as a proxy for the entropy and hence the level of self-organization in a power law system. Following from this, we see a strong correlation between  $B$  and  $b$  with  $\langle \ln E \rangle$  (Figure 3b) that is consistent with equation (18) above since  $B = 2/3b$  [Turcotte, 1997] and  $m$  is proportional to  $\ln E$  [Kanamori and Anderson, 1975]. Although  $b$  and  $B$  can depend on the catalogue and methods used to calculate them [Frohlich and Davis, 1993], the values of the exponents for mid-ocean ridges are still systematically higher ( $S$  systematically lower) than those for subduction zones. The results for the collision zones at present cannot be resolved with this degree of accuracy.

### 4.2. Data Reliability

[18] In order to verify if the results above may be skewed or influenced by the number of data points  $n$  available per zone [Kagan, 1997], Figure 4 shows a plot of  $n$  against  $b$  and  $B$  for all zones showing no significant dependence on  $n$  (slopes  $\sim 0$ ). This shows that the calculated scaling exponent values for different zones are not systematically influenced by data availability although the scatter roughly reduces with  $n$  as predicted by the central limit theorem. Also, in order to further confirm that the regional variation observed between the different tectonic regions is real, we perform a randomization of earthquake data between the different tectonic zones. This

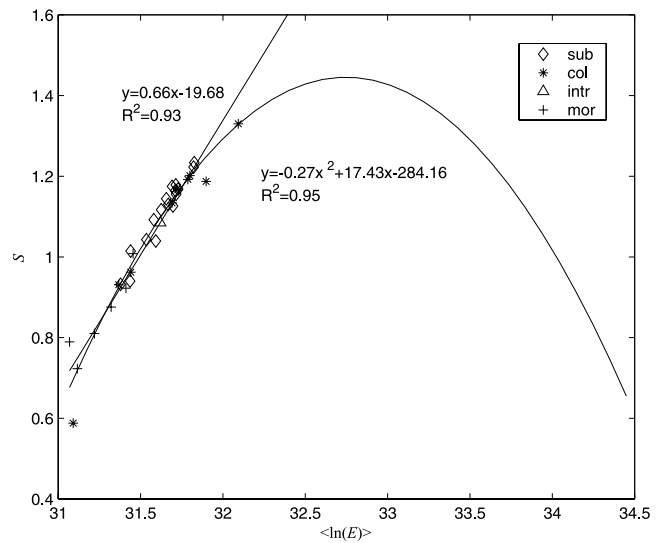
**Table 1.** Results of Parameters  $\langle \ln(E) \rangle$ ,  $S$ ,  $b$ , and  $B$  for Different Deformation Styles With Mean Values (Bold) and Standard Deviations (Parentheses)

FE	$n$	$\langle \ln(E) \rangle$	$S$	$b$ -MLF	$B$ -LSF
<i>Subduction Zones</i>					
1	340	31.71	1.16	0.91	0.63
5	249	31.82	1.22	0.85	0.54
6	200	31.67	1.13	0.93	0.66
7	83	31.58	1.09	0.98	0.68
12	927	31.44	1.01	1.08	0.75
13	367	31.38	0.93	1.13	0.81
14	595	31.83	1.23	0.85	0.64
15	579	31.69	1.18	0.91	0.69
16	318	31.71	1.18	0.90	0.76
18	232	31.72	1.16	0.90	0.68
19	606	31.66	1.14	0.93	0.62
20	107	31.59	1.04	0.97	0.64
21	121	31.70	1.13	0.91	0.64
22	395	31.73	1.17	0.90	0.68
23	422	31.53	1.04	1.02	0.64
24	423	31.62	1.12	0.96	0.63
46	83	31.44	0.94	1.08	0.75
Total	6047	<b>31.64 (0.13)</b>	<b>1.11 (0.09)</b>	<b>0.95 (0.08)</b>	<b>0.67 (0.06)</b>
<i>Collision Zones</i>					
25	53	31.79	1.19	0.86	0.61
26	62	31.90	1.19	0.85	0.52
27	52	31.44	0.96	1.08	0.81
29	107	31.80	1.20	0.86	0.57
30	108	31.73	1.17	0.90	0.51
31	52	31.37	0.93	1.15	0.70
41	57	32.09	1.33	0.74	0.45
47	33	31.09	0.59	1.46	0.70
48	93	31.69	1.13	0.94	0.66
Total	617	<b>31.65 (0.31)</b>	<b>1.08 (0.22)</b>	<b>0.98 (0.22)</b>	<b>0.62 (0.11)</b>
<i>Intracontinental</i>					
37	62	<b>31.62</b>	<b>1.08</b>	<b>1.00</b>	<b>0.61</b>
<i>Mid-Ocean Ridges</i>					
32	287	31.32	0.88	1.18	0.81
33	251	31.45	1.01	1.09	0.66
40	41	31.07	0.79	1.55	0.89
43	276	31.22	0.81	1.28	1.15
45	108	31.41	0.92	1.12	0.53
Total	963	<b>31.29 (0.16)</b>	<b>0.88 (0.09)</b>	<b>1.24 (0.19)</b>	<b>0.81 (0.24)</b>

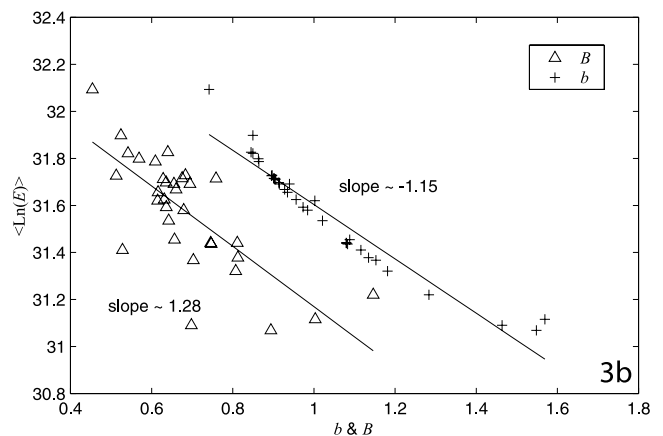
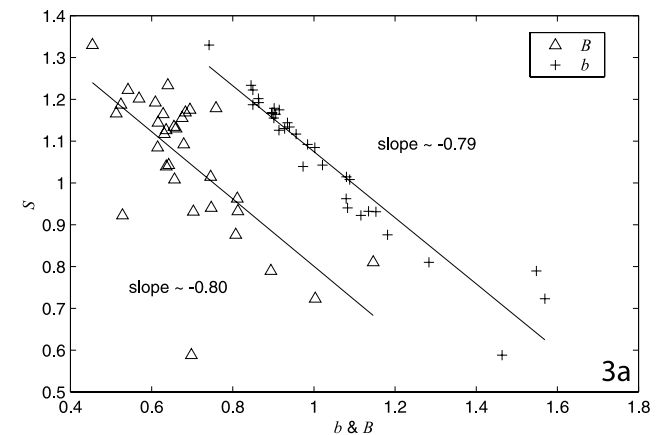
is done by randomly shuffling all magnitude data and keeping the positions of the events the same and then repeating the analysis as outlined above. For statistical stability, the shuffling is repeated 200 times. Table 2 shows the results of the randomization for three deformation styles and the mean standard deviation for the runs given in brackets. It can be seen that the variations between deformation styles disappear for the shuffled catalogues and that the parameters acquire values closer to global means similar to those found for a temporal ensemble [Main and Al-Kindy, 2002]. This is particularly true for mid-ocean ridges where  $\langle S \rangle$  increases from 0.88 for the unrandomized data to 1.01 for the randomized data and  $\langle b \rangle$  decreased from 1.24 to 0.98. This further confirms that mid-ocean ridges show a statistical deviation in their properties from those observed for continental settings contrary to the requirements of SOC, according to the criteria outlined above.

#### 4.3. Criticality

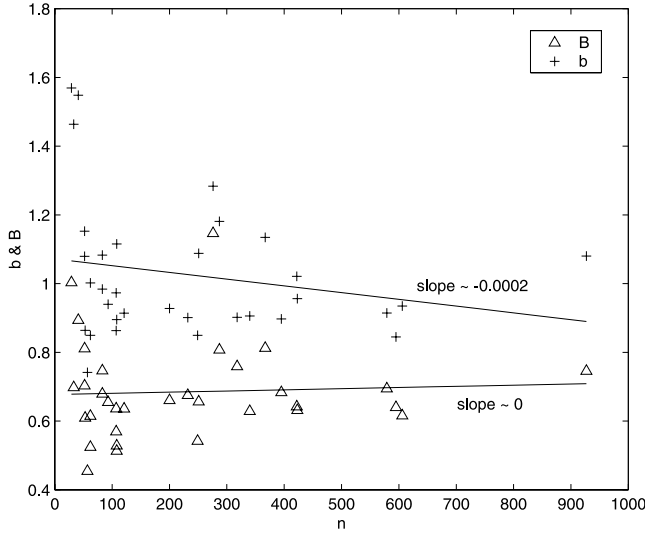
[19] It has been shown that equation (13) is linear for the case  $\theta \rightarrow \infty$  (critical). To establish if equation (13) holds for



**Figure 2.** Entropy energy phase diagram with best fit linear and quadratic fits to data (sub = subduction, col = collision, intr = intracontinental, and mor = mid-ocean ridge).



**Figure 3.** (a) Correlation of  $S$  with  $b$  and  $B$ . (b) Correlation of  $\langle \ln(E) \rangle$  with  $b$  and  $B$ . Note both correlations are negative.



**Figure 4.** Plot of exponents  $b$  and  $B$  as a function of the number of points  $n$  per FE region.

the regionalized data, a linear (critical) least squares fit is compared to a quadratic (intermittently critical) fit through the phase data. Figure 2 shows both a linear and quadratic fit with  $R$ -squared values of 0.93 and 0.95 respectively. A better fit (smaller residual error or higher  $R$ -squared) for a quadratic fit model is expected as it has an extra free parameter. We are therefore required to penalize for the extra free parameter in the quadratic fit in order to compare its fitness to a linear model. The penalization is done using Akaike's information criteria (AIC) [Akaike, 1978; Draper and Smith, 1998; Main et al., 1999]. If data points can be defined by

$$y_i = \gamma(x_i) + \varepsilon_i \quad (i = 1, 2, \dots, n), \quad (19)$$

where  $\gamma$  is the theoretical model,  $\varepsilon$  is an error term, and  $n$  is the number of points (here  $n = 32$ ), the residual sum of squares is then

$$S_R^2 = \sum_{i=1}^n [y_i - \hat{\gamma}(x_i)]^2, \quad (20)$$

with  $\hat{\gamma}$  the maximum likelihood model estimate and

$$L(y, x^*) = -\frac{n}{2} \ln(S_R^2). \quad (21)$$

**Table 2.** Comparison of Randomized With Unrandomized Data for Different Deformation Styles<sup>a</sup>

	Subduction Zones	Collision Zones	Mid-Ocean Ridges
<i>Unrandomized</i>			
$\langle \ln(E) \rangle$	31.64 (0.13)	31.73 (0.31)	31.35 (0.16)
$S$	1.11 (0.09)	1.14 (0.22)	0.90 (0.09)
$b$	0.95 (0.08)	0.92 (0.22)	1.17 (0.19)
$B$	0.67 (0.06)	0.60 (0.11)	0.78 (0.24)
<i>Randomized</i>			
$\langle \ln(E) \rangle$	31.60 (0.10)	31.59 (0.17)	31.59 (0.09)
$S$	1.09 (0.07)	1.06 (0.011)	1.09 (0.06)
$b$	0.98 (0.06)	1.00 (0.11)	0.98 (0.06)
$B$	0.67 (0.09)	0.60 (0.13)	0.66 (0.08)

<sup>a</sup>Standard deviations of parameters are given in brackets.

The Akaike information criterion is then

$$AIC = L(Y) - q, \quad (22)$$

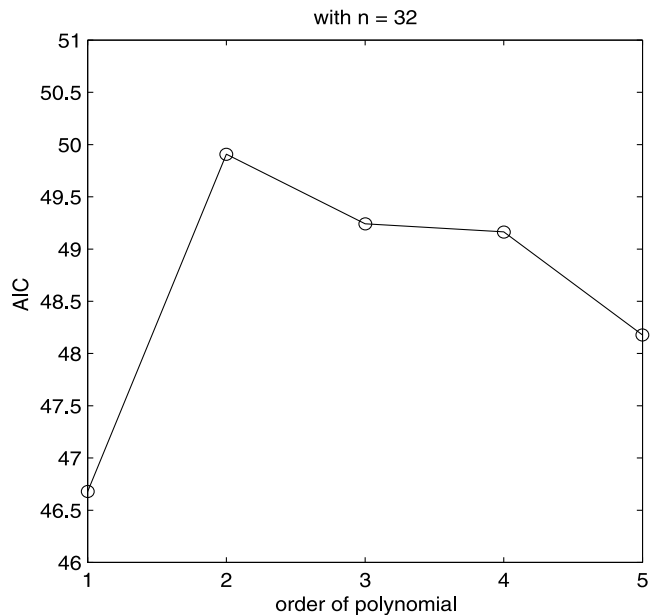
where  $q$  is the number of free parameters. Figure 5 shows that a quadratic fit gives a higher value for AIC when compared to a linear fit despite the penalty for the additional free parameter. The difference in AIC of 3.5 between the linear and quadratic fit is considered significant since  $AIC_{q+1} - AIC_q > 1$ . That is, by putting in an extra parameter AIC decreases by 1, but the likelihood increases by 4.5, so the resultant gain of 3.5 is significant. It can be concluded from Figure 5 that a quadratic fit statistically better describes the data than a linear fit. The sense of curvature is also similar to that predicted in Figure 1 from the analytical theory.

## 5. Discussion

### 5.1. Subcriticality

[20] The notion of criticality has been tested with reference to the requirements of SOC as outlined by Bak et al. [1987], in two ways: first, by investigating regional variations in earthquake parameter properties, and second, by referring to the theoretical predictions of statistical physics derived above. It has been seen from Figure 2 that the entropy-energy phase space for the various FE regions is projected along a statistically distinguishable curve rather than a straight line with subduction zones at the higher entropy level and mid-ocean ridges at the lower entropy level (Table 1). Subcriticality is therefore suggested in two ways:

[21] First, SOC requires that the global dynamics should be independent of local dynamics. This would require that the statistical properties of the system should be the same everywhere, implying there should not be any regional



**Figure 5.** Akaike information criterion (AIC) for polynomials of order 1, 2, 3, 4, and 5. The highest AIC (best fit) is for a second-order (quadratic) polynomial.



variations in the statistical properties observed, nor should the parameters rely on local mechanisms (subduction, spreading, etc.). Although it has been suggested that such regional variations might be the product of variations in data quality [Kagan, 1997], other studies have shown that local mechanisms such as slab temperature [Wiens and Gilbert, 1996] and spatial distribution of faults with plate deformation mechanism [Nicholson *et al.*, 2000] may influence the  $B$  value. As has been shown in Figure 3 above, the  $B$  and  $b$  values are closely related to the entropy  $S$  and therefore may be used as an analogue for it. There has also been no systematic correlation found between the amount of data available per region and the calculated  $B$  values as shown in Figure 4. As with Wiens and Gilbert [1996] and Nicholson *et al.* [2000], the results here also show regional variations in statistical properties but with the advantage of there being no possible retrospective bias in the method of data selection.

[22] Second, it is a prediction of equation (13) that for a strictly critical system, the entropy-energy phase space should be clustered at a point or plot along a straight line with very small fluctuations in  $S$  relative to  $\langle \ln E \rangle$ . It can be seen from Figure 2, however, that the phase space lies along a curve rather than a straight line, as confirmed quantitatively by AIC (Figure 5). The curvature indicates deviation from the prediction of equation (13) for a precisely critical state. The relatively large fluctuation in  $S$  along the  $y$ -axis is also indicative of a subcritical state. It must be stressed here that the observed curvature is strongly influenced by the presence of the extreme maximum and minimum points from the collision zone data. If the collision zone data are to be removed, the phase space may be better described as a straight line (SOC), although this would be considered a bias in the data selection. This conclusion nonetheless can be taken as a reinforcement of point 1 above and perhaps can be resolved in the future by analysis of larger catalogues.

## 5.2. Hallmarks of SOSC

[23] SOSC systems are unique in that they can organize themselves into a power law over a given region of the system rather than over the system as a whole, implying a finite correlation length, as expressed by the exponential term in the size-frequency gamma distribution. This finite correlation length can fluctuate with time and may intermittently reach a critical point when the resulting cluster spans the size of the region observed, producing a “large” earthquake. An SOSC state also has the advantage in that it does not rule out long or short-term observations that would otherwise be ruled out by a purely SOC state such as accelerated seismicity before large events [e.g., Zöller *et al.*, 2001] as well as regional variations in the  $B$  value both in space and time [Ogata *et al.*, 1991; Kagan, 1997; Wyss and Wiemer, 2000].

[24] What mechanisms, though, could be responsible for systematic regional statistical variations in the spatial ensemble? There are several candidates behind the variations observed. In reference to local dynamics, it has been found that heat flow, for example, plays an important role on the rheology of the crust [Ranalli, 1991] and therefore on seismicity [Sibson, 1982]. Given that heat flow varies systematically between tectonic regions worldwide, in particular when comparing oceanic to continental settings of

different age, thickness, etc. [Fowler, 1990], it is therefore not surprising that regional variations in the way the Earth self-organizes should exist. This is contrary to what is predicted from pure SOC models that have uniform and stationary boundary conditions. Another perhaps related mechanism is system energy dissipation or “seismic efficiency,” which at least for models of seismicity has been found to strongly affect  $B$  [Olami *et al.*, 1992] as well as  $\langle E \rangle$  [Main *et al.*, 2000]. These mechanisms will be examined more closely in a subsequent publication. Finally, it can be concluded from the myriad of possible statistical outcomes, as outlined in the regional study above, that SOSC may be a more flexible tool to describe a range of natural “power law” phenomenon including earthquakes. More important, SOSC suggests a degree of statistical predictability, at least spatially, in the population dynamics of the system whereas pure SOC does not. This gives an independent rationale for applying time-space dependent seismic hazard analysis to a population of events.

## 6. Conclusions

[25] In this paper we have shown, using the tools of statistical physics, the analysis of earthquake data from the CMT catalogue, and with reference to the original definition of SOC outlined by Bak *et al.* [1987], the following:

[26] 1. The entropy for a power law system such as the Earth’s crust is related to the expectation of the logarithm of the radiated energy where  $S = \Lambda + B \langle \ln E \rangle$  when  $\theta \rightarrow \infty$ . A system that deviates from this expression can be said to be subcritical or supercritical.

[27] 2. The analytical solutions show that for a power law system, the maximum possible entropy occurs at the critical point when  $B = 0$ . When  $B > 0$  ( $B < 0$ ), the critical point occurs below (above) the point of maximum entropy.

[28] 3. A good correlation was found between  $S$  and scaling exponents  $b$  and  $B$ . The exponents can therefore be said to be a good proxy for the entropy  $S$  and therefore the level of self-organization in a power law system.

[29] 4. Analysis of the regional CMT data shows subduction zones lie at the higher entropy regime (less ordered) with  $\langle S \rangle = 1.11$ , and mid-ocean ridges at the lower end (more ordered) with  $\langle S \rangle = 0.88$ , with collision-zones scattered between the extremes. This variation vanishes when the data are randomized, suggesting that the spatial fluctuations are significant. It was also found that the entropy energy phase space lies along a curve rather than a straight line, implying a state of SOSC. The curve is also in the same sense as that predicted from the analytical prediction from statistical physics for a degenerate set of energy levels.

[30] 5. SOSC is a better way of describing the regional variations in earthquake phenomenology as it can accommodate a range of observations including anomalous stress diffusion, accelerated seismicity, and regional variations in  $B$  and  $b$  that are otherwise ruled out by pure SOC. SOSC also suggests a degree of statistical predictability in earthquake population dynamics.

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